

Kepler, Harmony, and the Pythagorean Tradition

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Abstract

As important as Johannes Kepler's three laws of planetary motion are in the history of science, there is more to the story. Kepler's three famous laws were but a byproduct of his true mission, to hear the music of the universe.

While relating music and astronomy may seem strange to a modern reader, it makes sense when placed in the context of period music theory. In Kepler's time music theory covered three areas: *musica mundana*, *musica humana*, and *musica instrumentalis*. *Musica mundana* studied the process and harmony of the universe, *musica humana* unified and mediated between the human body and soul, and *musica instrumentalis* is music as performed by humans.

Universal harmony is an intellectual tradition linking back to Pythagoras. This paper will trace the lineage of this concept by weaving its thread through three major influences on Kepler – Pythagoras, Plato, and Ptolemy – and also on Kepler and his work. It concludes with a brief glance at vestigial remnants in Einstein. This paper will also investigate how the concept of harmony has unified arithmetic, geometry, music, and astronomy (known as the Greek quadrivium) since the origin of these disciplines.

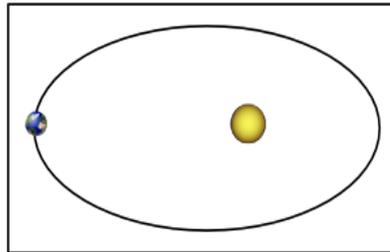
Kepler, Harmony, and the Pythagorean Tradition

Nothing holds me back. I am free to give myself up to the sacred madness... The die is cast, and I am writing the book – whether to be read by my contemporaries or by posterity matters not. Let it await its reader for a hundred years...¹

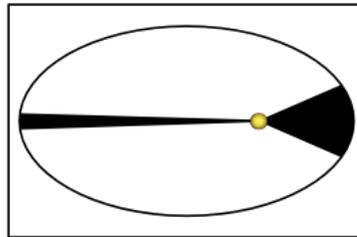
Johannes Kepler

To begin, I give away the ending: Kepler's three laws of planetary motion.

1. Each planet orbits on an ellipse with the sun on one focus. The other focus is a mathematical point.



2. An imaginary line drawn from the center of the sun to the center of a planet will sweep out equal areas in equal times.



3. $T^2=kD^3$ (T=period of revolution, D=mean distance from sun, and k=constant)

As important as these three laws are in the history of science, there is more to the story. Kepler's three famous laws were a byproduct of his true mission, to hear the music of the universe.²

¹ Johannes Kepler, "The Harmonies of the World," in *Great Books of the Western World* (Chicago: William Benton, 1952), p. 1010.

² Siglind Bruhn, *The Musical Order of the World* (Hillsdale, NY: Pendragon Press, 2005), p. 13.

While relating music and astronomy may seem strange to a modern reader, it makes sense when placed in the context of period music theory. In Kepler's time, music theory encompassed more than it does today. Music theory covered three areas: *musica mundana*, *musica humana*, and *musica instrumentalis*. *Musica mundana* studied the process and harmony of the universe, *musica humana* unified and mediated between the human body and soul, and *musica instrumentalis* is music as performed by humans.³

Why would Kepler presume that he, a mere human being, could possibly understand the music of the universe? Kepler, a devout Christian, reasoned that God created the universe to be aesthetically pleasing. Man, being a reflection of God, has similar tastes. Therefore, the music of the universe resembles that of man, and is comprehensible to man.⁴

Kepler took *musica mundana* literally. He believed that the planets performed a concert for inhabitants of the sun, or for a mind in the sun itself. The sun listened to the music, not with ears, but using a visual mechanism. "Of course," Kepler ponders, "it is not easy for us, dwelling on Earth, to conjecture what kind of vision might be in the Sun, what eyes, or what other instinct for perceiving these angles even without eyes, and for estimating the harmonies of the motions entering the vestibule of a Mind by some gate; what, finally, that Mind might be, in the Sun."⁵

While this, too, might seem strange to a modern reader, one must remember that the question of extraterrestrial life is as intriguing and unsolved today as it was in Kepler's time. On 6 March 2009 NASA launched the Kepler spacecraft to search for planets that might contain

³ Bruhn, p. 17.

⁴ Bruce Stephenson, *The Music of the Heavens* (Princeton, New Jersey: Princeton University Press, 1994), p. 7.

⁵ Kepler, p. 1080-1081.

life.⁶ Although life on or in the sun seems extremely unlikely, it may exist elsewhere in the universe. Kepler's conjecture, although off base in details, was ahead of its time in vision.

Concerning the Harmony of the Spheres, this is an intellectual tradition linking back to Pythagoras. This paper will trace the lineage of this concept by weaving its thread through three major influences on Kepler – Pythagoras, Plato, and Ptolemy – and also on Kepler and his work. It concludes with a glance at vestigial remnants in Einstein. At a higher level, this paper investigates how the concept of harmony has unified arithmetic, geometry, music, and astronomy (known as the Greek quadrivium) since the origin of these disciplines.⁷

Harmonia

In the beginning, harmony went by Harmonia, a goddess in the Olympian pantheon of ancient Greece. Her mother was Aphrodite, the goddess of love. Her father was Ares, the god of war. Harmonia excelled at reconciling opposites and creating balance.⁸

As Greek civilization shifted from narrative-based, anthropocentric mythos to logical, mechanistic logos, Harmonia shed her physical trappings and became a disembodied abstraction: harmony as universal ordering force.⁹ Pythagoras – the Greek mathematician, philosopher, music theorist, and cult leader – was influential in setting this course.¹⁰

⁶ nasa. Kepler *A Search for Habitable Planets*, March 6, 2009, http://www.nasa.gov/mission_pages/kepler/launch/index.html (accessed March 11, 2009).

⁷ Morris Kline, *Mathematics in Western Culture* (New York, Oxford University Press, 1953) p. 287

⁸ Robert E. Bell, *Women of Classical Mythology* (Santa Barbara, California, ABC-CLIO, 1991) p. 214

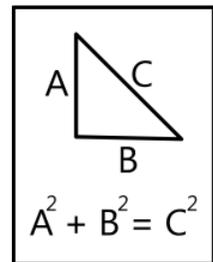
⁹ Richard Buxton, *From Myth to Reason?: Studies in the Development of Greek Thought* (New York, Oxford University Press, 1999), p. 25.

¹⁰ Bruhn, p.15.

Pythagoras

Pythagoras was born on the island of Samos around 580 B.C and lived to about 500 B.C. He was a traveling scholar, studying in Egypt, India, Miletus, Babylonia, and Italy. He founded a brotherhood known as the Pythagoreans. The Pythagoreans were secretive and observed many rituals. They did not wear clothing made of wool, eat meat or beans, touch a white rooster, stir a fire with iron, or leave marks of ashes on a pot.¹¹ New initiates were required to take a vow of silence for three years.¹² They devoted themselves to arithmetic, geometry, music, and astronomy. They conceptualized these disciplines as experiential conduits into universal harmony. Rigorous study would promote spiritual resonance with the universe and escape from the cycle of rebirth.

Pythagoreans worshiped numbers as the fundamental basis of all reality. Among mathematicians, Pythagoras is known for the Pythagorean Theorem. It is said that Pythagoras was so excited by its discovery that he sacrificed an ox to celebrate.¹³



Among music students, Pythagoras is known for the Pythagorean tuning system. Legend has it that Pythagoras walked by a blacksmith shop and noticed hammers of different weights sounding different pitches.¹⁴ It is likely that he experimented with a monochord to determine exact mathematical proportions. Pythagoras discovered that all musically perfect intervals can be constructed using numerical proportions built with the numbers one, two, three, and four.

1:2=P8
2:3=P5
3:4=P4

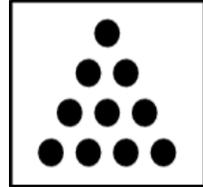
¹¹ Morris Kline, *Mathematics in Western Culture*, p. 40-41.

¹² Bruhn, p. 65.

¹³ Morris Kline, *Mathematics for the Nonmathematician* (New York, Dover Publications, 1967), p. 66

¹⁴ Andre Barbera *Pythagoras Grove Music Online. Oxford Music Online.* [Online][Cited: 16 February 2009]. <http://www.oxfordmusiconline.com/subscriber/article/grove/music/22603>

The Pythagoreans held this set of numbers in high esteem. They formed what the Pythagoreans called the tetraktys. New initiates had to take an oath: “Bless us, divine number, thou who generate gods and men! O holy, holy tetraktys, thou that contained the root and source of eternally flowing creation!” The significance of these numbers may originate in the ancient belief that all matter is constructed from four elements: earth, air, fire, and water.¹⁵ The tetraktys was symbolized as a triangle of ten dots.¹⁶



The Pythagoreans taught that planets produced musical tones as they moved. This may be rooted in the sound an object makes while being spun on string. The Pythagoreans might have reasoned that planets speeding through the heavens must also make sounds.¹⁷ These cannot be heard, rationalized the Pythagoreans, because they have always existed and are therefore impossible to notice.¹⁸ In modern psychological language one would suggest that the sounds exist, but humans are completely habituated to them.¹⁹

¹⁵ Morris Kline, *Mathematics and the Physical World* (New York, Thomas Y. Crowell Company, 1959), p. 29.

¹⁶ Bruhn, p. 65.

¹⁷ Morris Kline, *Mathematics in Western Culture*, p. 76.

¹⁸ Stephenson, p. 16.

¹⁹ "Animal behavior," *Encyclopaedia Britannica*. 2009, <http://search.eb.com/eb/article-48654> (accessed March 12, 2009).

Plato

Plato was born about seventy years after the death of Pythagoras.²⁰ Plato was friend and unofficial student of Socrates, and teacher to Aristotle.

The legacy of universal harmony is apparent in his dialogues. In the *Timaeus*, Plato writes of a hero, Er, killed in battle. Twelve days later, Er awoke on the funeral pile with a vision from the afterlife. He described a sky filled with massive shafts, hooks, and spindles turning eight concentric planets through the heavens. A solitary siren rides on every planet singing a single pitch.²¹ This is likely a coded message in the Pythagorean tradition. Plato was respecting the Pythagorean oath of secrecy by cloaking the theory in mythology.²²

Plato, like Pythagoras, used harmony to tie together music and astronomy:

Motion appears in many respects – but there are two kinds, one which appears in astronomy and another which is the echo of that. As the eyes are made for astronomy so are the ears made for the motion which produces harmony: and thus we have two sister sciences, as the Pythagoreans teach, and we assent.²³

Plato advocated the synthesis of music and mathematics. In *The Republic*, Plato admires the Pythagoreans use of mathematical proportions in determining music consonance. In the *Philebus* he speaks of dissonance being made “commensurable and harmonious by introducing the principle of number.”²⁴

²⁰ Warren Anderson and Thomas J Mathiesen, “Plato,” *Grove Music Online. Oxford Music Online*. [Online][Cited: 21 February 2009.]<http://www.oxfordmusiconline.com/subscriber/article/grove/music/21922>.

²¹ Stephenson, p. 17.

²² Bruhn, p. 14.

²³ Morris Kline, *Mathematics for the Nonmathematician*, p. 436.

²⁴ Warren Anderson and Thomas J Mathiesen, “Plato,” *Grove Music Online. Oxford Music Online*. [Online][Cited: 21 February 2009.]<http://www.oxfordmusiconline.com/subscriber/article/grove/music/21922>

Plato disagreed with the Pythagoreans on hierarchy. He believed the harmony of mathematics to be superior to the harmony of music. He criticized the Pythagoreans for focusing too much on the actual sound of musical intervals, and not concentrating enough on the silent harmony of pure numbers.

Plato, like Pythagoras, considered mathematics to be a spiritual experience. Plato believed that the physical world is but an imperfect and temporary representation of an ideal world of the abstract divine. Our senses are limited to the physical world. Only the mind can know the perfect eternal. Man must train himself to this goal. Plato believed mathematics to be the proper tool for this task. The world of mathematics is true reality, and the physical world is but a shadow of this ideal.²⁵ Plato says of mathematicians:

And do you not know also that although they make use of the visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter... they are really seeking to behold the things themselves which can be seen only with the eye of the mind?²⁶

This duality is represented in the allegory of the cave. A prisoner spends his entire life chained to the wall of a cave. He observes shadows projected on the opposite wall. He believes this to be the sum of reality. It is only after leaving the cave that the prisoner does experience true reality. The cave is a metaphor for the everyday physical world. One must study mathematics to be released into the sunlight.²⁷

²⁵ Morris Kline *Mathematics in Western Culture*, p. 78.

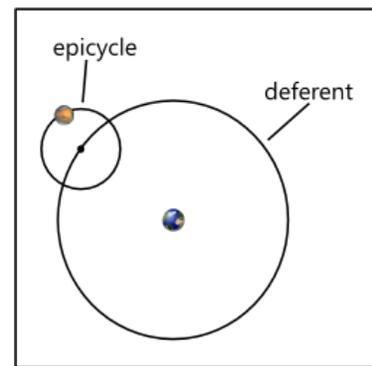
²⁶ Morris Kline, *Mathematics for the Nonmathematician* p. 32.

²⁷ Morris Kline *Mathematics in Western Culture*, p. 32.

Ptolemy

About five hundred years later, another great thinker devoted himself to music, mathematics, and astronomy. Ptolemy worked in Alexandria during the second century A.D.²⁸ He wrote a major book on astronomy, *He mathematika syntaxis* (The Mathematical Collection), now generally known as the *Almagest*.²⁹ He also wrote an important book on music theory, *Harmonika*.³⁰

The *Almagest* was an overview of Greek astronomy up to that time. It describes a geocentric (earth centered) universe. The planets were thought to circle the earth in an orbit called the deferent. Each planet rotated on a smaller circle called the epicycle. This elaborate system was needed to explain the complicated orbits (the word 'planet' means 'wanderer' in Greek)³¹ observed from the earth. Though this system was actually conceived by an earlier thinker, Hipparchus, it has become known as the Ptolemaic model of the universe.³²



The *Almagest* advanced the cutting edge of mathematics. While attempting to measure the sizes and distances of the planets, Ptolemy set the form for trigonometry that would be maintained for over a thousand years.³³

²⁸ Luckas Richter, "Ptolemy," *Grove Music Online. Oxford Music Online*, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/22510> (accessed February 21, 2009).

²⁹ Morris Kline, *Mathematics for the Nonmathematician*, p. 158.

³⁰ Stephenson, p. 32-33.

³¹ Morris Kline, *Mathematics for the Nonmathematician*, p. 190.

³² Morris Kline *Mathematics in Western Culture*, p. 83-84.

³³ Morris Kline *Mathematics in Western Culture*, p. 84.

In his other major work, *Harmonika*, Ptolemy discussed both the Pythagorean and Aristoxenian schools. Whereas Pythagoras treated music and mathematics as being intimately related, Aristoxenus believed music to be a free standing structure³⁴ completely separate from both mathematics and astronomy.³⁵

Ptolemy took a reconciliatory approach to the empirical versus mathematical debate.³⁶ He used a monochord to experiment with the mathematics of musical intervals. Ptolemy's empiricism manifests itself in his use of the pure third.³⁷

$$4:5 = M3$$

In the *Harmonika*, Ptolemy expressed a familiar Pythagorean belief. He reasoned that harmony is an ordering principle, a force of reason. Each of the two highest senses, sight and hearing, give rise to a field of science by observing harmony. The best place to observe harmony with the eyes is in astronomy. The best way to observe harmony with the ears is music. It is the concept of harmony that unifies these two fields.

Ptolemy also contemplated various ways music relates to the planets. Some of the systems are based on astrology: various planets have musical characteristics based on their astrological significance. Others are based on astronomical observations: music rises in pitch as a planet rises from the horizon, and falls in pitch as a planet descends.³⁸

³⁴ Stephenson, p. 33.

³⁵ Annie Belis, "Aristoxenus," *Grove Music Online. Oxford Music Online*. [Online][Cited: 7 March 2009.]<http://www.oxfordmusiconline.com/subscriber/article/grove/music/01248>.

³⁶ Stephenson, p. 33.

³⁷ Luckas Richter "Ptolemy," *Grove Music Online. Oxford Music Online*.

³⁸ Stephenson, p. 35-36.

Fifteen hundred years later, Kepler felt a kinship with Ptolemy. While disagreeing on specific content, he was encouraged by the similarity of their quests. Kepler had this to say about Ptolemy's *Harmonika*:

There, beyond my expectation and with the greatest wonder, I found approximately the whole third book given over to the same consideration of celestial harmony, fifteen hundred years ago. But indeed astronomy was far from being of age as yet; and Ptolemy, in an unfortunate attempt, could make others subject to despair, as being one who, like Scipio in Cicero, seemed to have recited a pleasant Pythagorean dream rather than have aided philosophy. But both the crudeness of the ancient philosophy and this exact agreement in our meditations, down to the last hair, over an interval of fifteen centuries, greatly strengthened me in getting on with the job.³⁹

Kepler

Johannes Kepler (1571-1630) was born in Weil der Stadt, a small town in Germany. His father was a mercenary and left when Kepler was a boy. The best profession at that time for an individual of intelligence and poverty was the clergy. Kepler tested for and received a scholarship to train⁴⁰ and while at school studied music theory and sung four-part psalms and hymns.⁴¹ At age seventeen he earned a Bachelor's degree.⁴² He continued to study and earned a Master's degree in 1591. In 1594 he took a job as a professor of Mathematics and Morals at the University of Gratz.⁴³

³⁹ Kepler, p. 1010.

⁴⁰ Bruhn, p. 20-22.

⁴¹ Susi Jeans and H.F. Cohen. "Johannes Kepler," *Grove Music Online. Oxford Music Online.*
<http://www.oxfordmusiconline.com/subscriber/article/grove/music/14903> (accessed March 12, 2009).

⁴² Bruhn, p. 21.

⁴³ Susi Jeans and H.F. Cohen, "Johannes Kepler"

His new position required him to study astrology along with astronomy. He resigned himself to casting horoscopes as a way to pay the bills. He was known to refer to astrology as the daughter of astronomy who nursed her own mother.⁴⁴

At Graz, he published his first treatise, the *Introduction to the cosmographic treatise, containing the cosmographic mystery concerning the remarkable proportions of the heavenly spheres, and concerning the genuine and proper causes of the number, magnitude, and periodic motions of the spheres, demonstrated by means of the five regular geometric solids.*⁴⁵ Today it is referred to as the *Cosmographic Mystery* (1597).⁴⁶

Kepler proposed a polyhedral theory in which the planets rotated on spheres separated by one each of the five regular polyhedra. In the *Cosmographic Mystery* Kepler writes:

I undertake to prove that God, in creating the universe and regulating the order of the cosmos, had in view the five regular bodies of geometry as known since the days of Pythagoras and Plato, and that he has fixed according to those dimensions, the number of heavens, their proportions, and the relations of their movements.⁴⁷

Although Kepler was very fond of this theory, it contradicted the data. Kepler believed better measurements would solve the problem. In 1600 Kepler moved to Prague to assist the astronomer Tycho Brahe. Kepler hoped Brahe's precise information could prove his theory right. Kepler got more than he bargained for when a year later Brahe died and Kepler took over the whole operation.⁴⁸

In 1609 Kepler published *On the Motions of the Planet Mars*. Kepler had discarded the polyhedral theory and replaced it with the first two of his three laws of planetary motion.

⁴⁴ Morris Kline, *Mathematics in Western Culture*, p. 113.

⁴⁵ Stephenson, p. 75.

⁴⁶ Bruhn, p. 21.

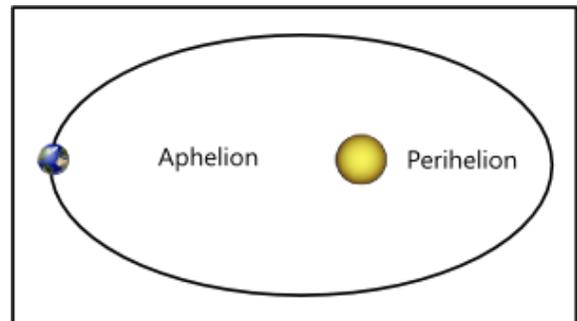
⁴⁷ Morris Kline, *Mathematics in Western Culture*, p. 113-114.

⁴⁸ Stephenson, p. 98-99.

These laws are illustrated at the beginning of this text. In 1619 Kepler completed his most famous work, *Harmonices mundi libri* (The Harmony of the World).⁴⁹ This book contains Kepler's third law of planetary motion. This law is also shown at the beginning of this text.

The Harmony of the World

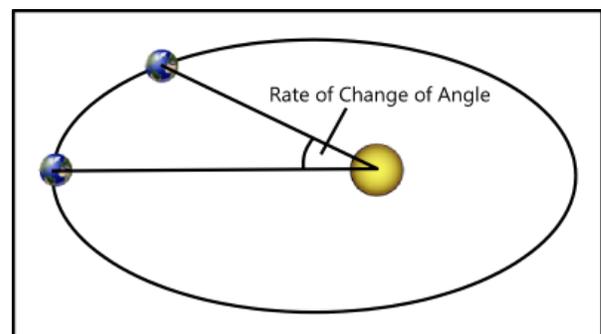
To begin, a brief review of essentials from *On the Motions of the Planet Mars*: Planets spin around the sun in an ellipse. As they approach the sun, they speed up. As they move farther from the sun, they slow down. The perihelion is the point in



the orbit that is closest to the sun, when the planet is moving fastest. The aphelion is the point of farthest distance, when the planet is moving slowest.⁵⁰

In *The Harmony of the World* Kepler theorized on where and how the Creator would choose to notate his music. Kepler considered and eliminated options until one remained. The Creator would embed his harmony in moments of extreme motion. Therefore, Kepler looked to the aphelion and perihelion.⁵¹

Kepler decided against straightforward distance over time. He chose angular velocity as being more perceptible to an observer in the sun. Kepler writes, "For what purpose would there be



⁴⁹ Bruhn, p. 22.

⁵⁰ Morris Kline, *Mathematics for the Nonmathematician*, p. 328-330.

⁵¹ Stephenson, p. 146.

harmonies among the paths; indeed who would perceive these harmonies?"⁵² Kepler computed these as average daily angular velocities as he had no mathematics to deal with instantaneous velocity; Newton and Leibnitz would not invent the Calculus for another fifty years.⁵³

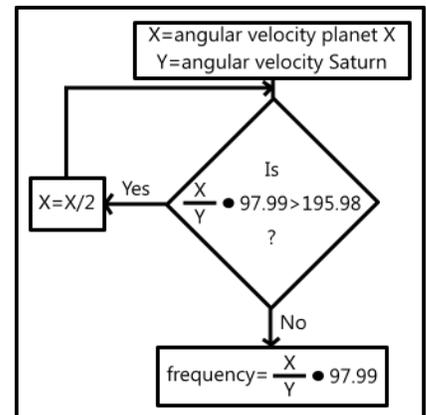
The next step is to convert these angular velocities into musical intervals. Kepler used the technique of numerical proportions developed by Pythagoras.

As far as Kepler knew, Saturn was the outermost planet. Therefore, he assigned Saturn to the denominator and each of the other planets, in turn, to the numerator. He did this for both aphelion and perihelion.⁵⁴

$\frac{\text{aphelion of planet X}}{\text{aphelion of Saturn}}$	$\frac{\text{perihelion of planet X}}{\text{aphelion of Saturn}}$
$\frac{\text{aphelion of planet X}}{\text{perihelion of Saturn}}$	$\frac{\text{perihelion of planet X}}{\text{perihelion of Saturn}}$

Kepler then set about using these proportions to build an octave scale. To do this he would need to compress all the intervals into an octave range.

Kepler established a base pitch of 97.99 hertz (G_2) and assigned this to Saturn. Doubling this frequency raises the pitch by an octave. Therefore, the pitches generated by the proportions had to be less than or equal to 195.98 (97.99 multiplied by 2). If a proportion multiplied by 97.99 resulted in a frequency above



195.98, Kepler divided the angular velocity in half until the frequency did not exceed 195.98.⁵⁵

The reader should recall that halving the frequency lowers a tone by one octave.⁵⁶

⁵² Stephenson, p. 148.

⁵³ Morris Kline, *Mathematics for the Nonmathematician*, p. 336.

⁵⁴ Stephenson, p. 154-158.

⁵⁵ Stephenson, p. 154-158.

⁵⁶ James Jeans, *Science & Music* (New York: Dover Publications, 1968), p. 154.

Kepler then used the adjusted proportions to compute four different pitches for each planet. He rounded each frequency to the nearest pitch using just intonation.⁵⁷ Kepler did not specify an exact tolerance range for pitches.⁵⁸ Kepler's outcomes are summarized in the following chart reproduced from Bruce Stephenson's book, *The Music of the Heavens*.⁵⁹

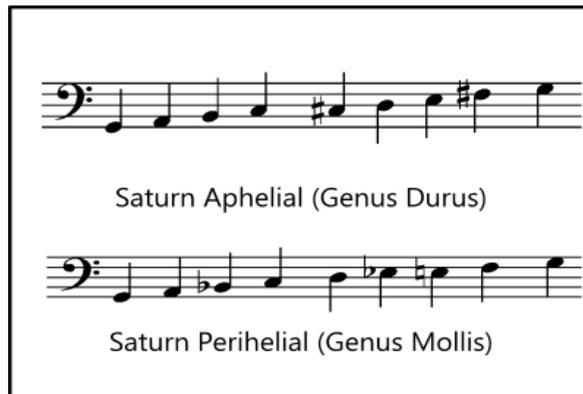
Extreme Motion		Saturn's aphelial motion=G		Saturn's perihelial motion=G	
4'30"	Jupiter aphelial			G	4'30"
3'49"	Earth's perihelial			E	3'35"
3'34"	Earth's aphelial	G	3'34"	D#	3'36"
3'17"	Mars's aphelial	F#	3'21"	D	3'23"
3'3"	Venus's perihelial			C	3'0"
3'0"	Mercury perihelial	E	2'58"		
2'58"	Venus's aphelial	E	2'58"		
2'45"	Jupiter's perihelial	D	2'41"	Bb	2'42"
2'34"	Mercury aphelial	C#	2'30"	A	2'32"
2'23"	Mars's perihelial	C	2'23"		
2'15"	Jupiter's aphelial	B	2'14"		
2'15"	Saturn's perihelial	B	2'14"	G	2'15"
1'46"	Saturn's aphelial	G	1'46"		

⁵⁷ Stephenson, p. 119.

⁵⁸ *Ibid.*, p. 147.

⁵⁹ *Ibid.*, p. 158.

Kepler derived two scales from the notes. Assigning the base pitch to Saturn’s aphelion generates genus durus. Durus means “hard” and is similar to our modern major scale. Placing the base pitch on Saturn’s perihelion generates genus mollis. Mollis means “soft” and resembles our modern minor scale.⁶⁰



In the epilogue of *Harmony of the World*, Kepler hypothesizes a symbiotic relationship between the planets and the sun:

Not only does light go out from the sun into the whole world, as from the focus or eye of the world, as life and heat from the heart, as every moment from the King and mover, but conversely also by royal law these returns, so to speak, of every lovely harmony are collected in the sun from every providence in the world, nay, the forms of movements by twos flow together and are bound into one harmony by the work of some mind.⁶¹

Thinkers in the seventeenth and eighteenth century thought they had it all figured out. Kepler decoded planetary motion. Galileo figured out falling bodies and projectile motion. Newton, in 1687, unified Kepler and Galileo with the universal law of gravitation.⁶² Mankind seemed only a small step from knowing everything.⁶³

⁶⁰ Stephenson, p. 156-157.

⁶¹ Kepler, p. 1081.

⁶² Morris Kline, *Mathematics in Western Culture*, p. 196-213.

⁶³ *Ibid.*, p. 239.

Einstein

Einstein did to Newton what Kepler did to Ptolemy. Albert Einstein tore down Newton's universe and built his own. He designed a four-dimensional non-Euclidian geometry where the shortest path between two points in Plato's ideal world projects its shadow on the cave wall as an unseen force called gravity.⁶⁴

Almost as shocking as discovering that one's home planet (earth) is not the privileged unmoving center of the universe, is the knowledge that space and time are not absolute. Although Einstein's theory was radically new, Einstein himself was solidly in the Pythagorean tradition.

Music was very important to Einstein. He began playing the violin at age six and became a competent and passionate violinist.⁶⁵ One of his teachers said of young Einstein, "One student, by the name Einstein, even sparkled by rendering an adagio from a Beethoven sonata with deep understanding."⁶⁶ As an adult, playing the violin helped him think. When working on a difficult problem he would often improvise on the violin late into the night. A friend recalled how Einstein would call out "I've got it!" when struck with insight "as if by inspiration, the answer to the problem would have come to him in the midst of music."⁶⁷

⁶⁴ Morris Kline, *Mathematics in Western Culture*, p. 419-452.

⁶⁵ Alice Calaprice, *The Expanded Quotable Einstein* (Princeton, New Jersey: Princeton University Press and Hebrew University of Jerusalem, 2000), p. 153.

⁶⁶ Walter Isaacson, *Einstein His Life and Universe* (New York, NY: Simon & Schuster, 2007), p. 29.

⁶⁷ *Ibid.*, p. 14.

Einstein tried to create the same kind of purity and simplicity in his mathematics and physics as he found in Mozart's compositions. Einstein says, "Mozart's music is so pure and beautiful that I see it as a reflection of the inner beauty of the universe."⁶⁸

Walter Isaacson says of Einstein's relationship with music:

It was not so much an escape as it was a connection: to the harmony underlying the universe, to the creative genius of the great composers, and to other people who felt comfortable bonding with more than just words. He was awed, both in music, and in physics, by the beauty of harmonies.⁶⁹

One can imagine Einstein in his kitchen, bowing his violin, pondering the heavens, connected by some twisted time-space geometry to Pythagoras, sitting cross-legged under an olive tree, experimenting with his monochord, also pondering the heavens.

And as always, the orchestra of the universe plays along.

⁶⁸ Calaprice, p. 157.

⁶⁹ Isaacson, p. 37.

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